A graphical illustration of optimal monetary policy in the New Keynesian framework

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Abstract

The canonical New Keynesian model with optimal monetary policy under discretion provides a simple theoretical foundation for *Flexible Inflation Targeting*. A graphical illustration of the monetary policy problem in this framework is provided to facilitate intuition.

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1 Introduction

The canonical New Keynesian model with optimal monetary policy under discretion provides a simple theoretical foundation for *Flexible Inflation Targeting* (see e.g. Svensson, 2010). The theoretical framework is derived and analysed in detail by Woodford (2003) and in the textbook by Galí (2008). See also the early overview by Clarida et al. (1999). Insights from the theoretical analysis has informed monetary policy in practice and has shaped the communication of trade-off management for monetary policymakers in the form of illustrative graphs of economic outcomes and forecasts, see in particular Qvigstad (2006), Walsh (2014), Haldane and Qvigstad (2015), and Carney (2017). This note develops the graphical representation of the optimal monetary policy

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problem in the basic New Keynesian model to facilitate intuition as well as teaching of monetary policy in theory and practice. For similar graphical approaches, see Carlin and Soskice (2005) and Benigno (2015). Section 2 briefly lays out the textbook theory, while the graphical illustration is given in 3. Section 4 contains a few concluding remarks.

2 The theory

In addition to a specification of monetary policy, the canonical New Keynesian model consists of the two equations

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \tag{1}$$

$$x_t = E_t x_{t+1} - \frac{1}{\varsigma}\left(i_t - E_t \pi_{t+1} - r_t^*\right) \tag{2}$$

where E_t is the expectations operator, π_t is inflation at time t in deviation from its target π^*, x_t is the output gap defined as output in deviation from its efficient level, and i_t is the nominal interest rate in deviation from its normal deterministic steady-state value i^*. The first equation is the New Keynesian Phillips curve, and the second is the forward-looking IS curve. The canonical model is derived from its microfoundations by Woodford (2003) and Galí (2008) among others.

There are two shock processes in the model. u_t is a cost-push process, and r_t^* is the efficient equilibrium real interest rate in deviation from its steady state level $r^* = i^* - \pi^*$. For simplicity, let the two shock processes follow independent stochastic processes

$$r_t^* = \mu_r r_{t-1}^* + \nu_{r,t} \tag{3}$$

$$u_t = \mu_u u_{t-1} + \nu_{u,t} \tag{4}$$

where $\nu_{r,t} \sim N(0, \sigma_r^2)$ and $\nu_{u,t} \sim N(0, \sigma_u^2)$.

Under optimal discretionary policy, a policymaker minimises the period loss function

$$L = \pi_t^2 + \lambda x_t^2 \tag{5}$$

each period subject to the Phillips curve in (1) while taking expectations as given. This gives rise to the targeting rule

$$\pi_t = -\frac{\lambda}{\kappa} x_t \tag{6}$$

stating the optimal policy trade-off between inflation and the output gap. The interest rate consistent with this optimal allocation can now be found from the IS curve in (2). For details, see Galí (2008).

The condition in (6) is the 'golden rule' of monetary policy strategy. Following this rule, whenever a trade-off between inflation and the outgap arises, monetary policy stimulates or contracts the economy so as to ensure that the deviation of output from its potential and the deviation of inflation from target have opposite signs. The targeting rule is steeper, the higher the weight on the output gap (higher λ), and the flatter the Phillips curve (lower κ). When the targeting rule is steep, monetary policy allows inflation to absorb more—and the output gap less—of the adjustment after cost-push shocks.

3 Graphical illustration

The remit for monetary policy is to minimise the loss function (5). The first-best outcome is for inflation to be on target while the output gap is closed so that $\pi = x = 0$ at all times.[1] The loss is zero in this case. Deviations of inflation from target and of output from potential increase the loss depending on the shape of the loss function.

The first row in Figure 1 shows graphs of the loss function in (5) for $\lambda = 0$ (left panel), $\lambda = 0.5$ (middle panel) and $\lambda = 1$ (right panel). When $\lambda = 0$, the policymaker is an "inflation nutter" (King, 1997) who only cares about inflation. In this case, losses increase in the deviation of inflation from target, but not in the output gap from zero. In general, when $\lambda > 0$, losses increase both in deviations of inflation from target and in the output gap from zero. When $\lambda = 1$, the deviations in the two target variables are equally costly, and the graph is a paraboloid. Since the loss function is quadratic in its arguments, small deviations from target incur very small losses, while large deviations are very costly.

The second row in Figure 1 shows projections of the loss functions onto the $\{x, \pi\}$ plane. These isoloss curves represent combinations of output and inflation gaps that give the same loss, increasing in the distance from the origin where $\{x, \pi\} = \{0\%, 2\%\}$ implies a loss of zero ($L = 0$). When $\lambda = 1$ and the loss function is symmetric, the isoloss curves are circles in $\{x, \pi\}$. At the other extreme, when $\lambda = 0$, the isoloss curves are straight horizontal lines. For values of $\lambda \in (0, 1)$, the curves are ellipses, and the policymaker would trade larger output gaps for given deviations of inflation from target.

The monetary policymaker cannot simply choose any point on the $\{x, \pi\}$ plane. In the short-run—for as long as expectations can be taken as given—the policymaker can set the nominal interest

[1] Time subscripts t are suppressed in this section for simplicity.

rate, and so by (2) the output gap, so as to choose a point on the Phillips curve given by (1). Figure 2 shows the Phillips curve in $\{x, \pi\}$ space for two different values of $\kappa \in (0, 1)$ along with an isoloss curve representing a strictly positive loss for the case when $\lambda = 1$. Inflation expectations are taken as given and, without loss of generality, is set to the inflation target. The flatter the Phillips curve, the higher the sacrifice ratio in the sense that larger deviations in the output gap are needed to bring about a given change in inflation.

In Figure 2, the first-best policy can always be achieved regardless of the slope of the Phillips curve.[2] By setting the interest rate such that the output gap is zero, monetary policy also prevents inflation from moving away from target, and the loss is zero. Specifically, any shock to r^* in (2) that would move the economy along the Phillips curve without a monetary policy response (e.g. to a point on the isoloss curve drawn) can be neutralised by a corresponding nominal interest rate.

Now suppose that the economy is exposed to a positive cost-push shock, u, as shown in Figure 3. This shock shifts the Phillips curve upwards in $\{x, \pi\}$ space. If monetary policy does not respond, the output gap will remain closed and inflation will increase to a point on a high level curve for the loss function. At the other extreme, if monetary policy offsets the effect of the shock on inflation completely by inflicting a recession on the economy, the loss from inflation will be zero. But depending on the slope of the Phillips curve, the loss from the output gap will now be high.

The optimal monetary policy response is to tighten policy enough to move the economy down the Phillips curve to the point where the loss is minimised. This $\{x, \pi\}$ combination is a tangency point with the isoloss curve representing the lowest achievable loss given the structure of the economy as summarised in the Phillips curve. When the Phillips curve is flat, the sacrifice ratio is high, and monetary policy is limited in its ability to reach lower isocost curves. The policymaker therefore lets inflation absorb more of the adjustment to the shock.

Figure 3 shows the targeting rule in (6) as the line through the tangency points for all possible realisations of the cost-push shock. The targeting rule shows the optimal balance between inflation and output gap deviations following tradeoff-inducing disturbances to the economy. Optimal monetary policy under discretion keeps the economy on this downward-sloping targeting rule at all times, where the actual position on the targeting rule depends on position of the Phillips curve.[3] If realisations of the output gap and inflation fall below the targeting rule, monetary policy has been too tight. If observations fall above the targeting rule, it has been too loose. More generally, realisations should, at least, fall in the North-West and South-East quadrants.

[2]This result does require that inflation expectations are on target. With a non-zero expectation term, the Phillips curve would not be centered at the origin.

[3]Note that in general the position of the Phillips curve will also be determined by expectational dynamics.

4 Concluding remarks

If monetary policy is conducted under commitment, so that the policymaker actively seeks to affect private sector expectations, the simple targeting rule in (6) no longer represents the optimal trade-off between inflation and the output gap. However, if commitment is from a timeless perspective as advocated by (Woodford, 1999), optimal policy can be represented by a version of the rule where the output gap is replaced by the change in the output gap. The graphical analysis remains the same in this case with the first difference of the output gap replacing the output gap itself on the x-axis.

When monetary policy is constrained by an effective lower bound for the nominal interest rate, the optimal balance between output and inflation stabilisation in (6) may be infeasible. If the interest rate required to keep the economy on the targeting rule falls below the lower bound, the best monetary policy can do is to set the interest rate at the lower bound (see e.g. Nakov, 2008).

Deviations from the simple targeting rule will be desirable if the loss function includes variables other than inflation and the output gap. For example, a role for interest rate smoothing can be introduced by introducing a quadratic term in interest rate changes. Isocost curves would now be spheres in $\{x, \pi, \Delta i\}$ space.

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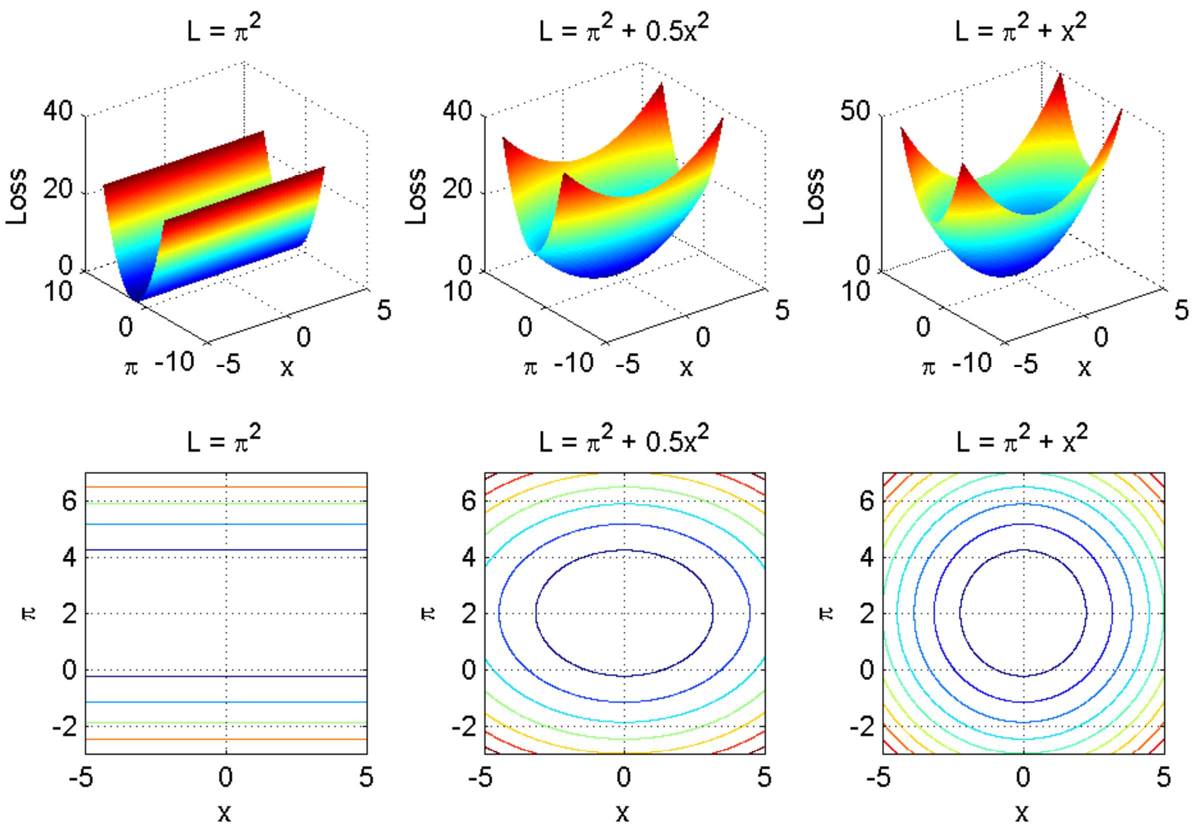

Figure 1: Graphs (upper row) and level curves (lower row) of the loss function in (5) for $\lambda = 0$ (left panels), $\lambda = 0.5$ (middle panels) and $\lambda = 1$ (right panels)

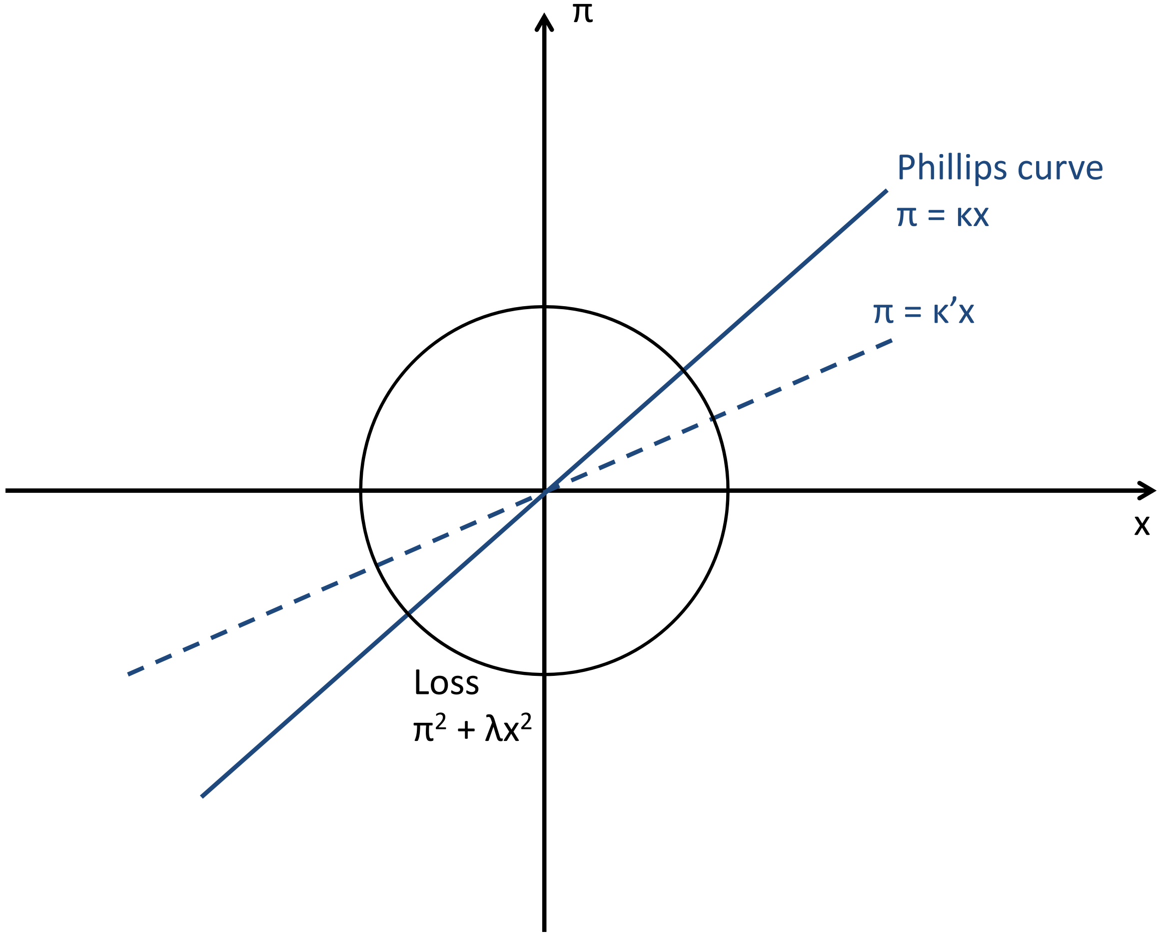

Figure 2: Graphical illustration of optimal monetary policy under discretion I

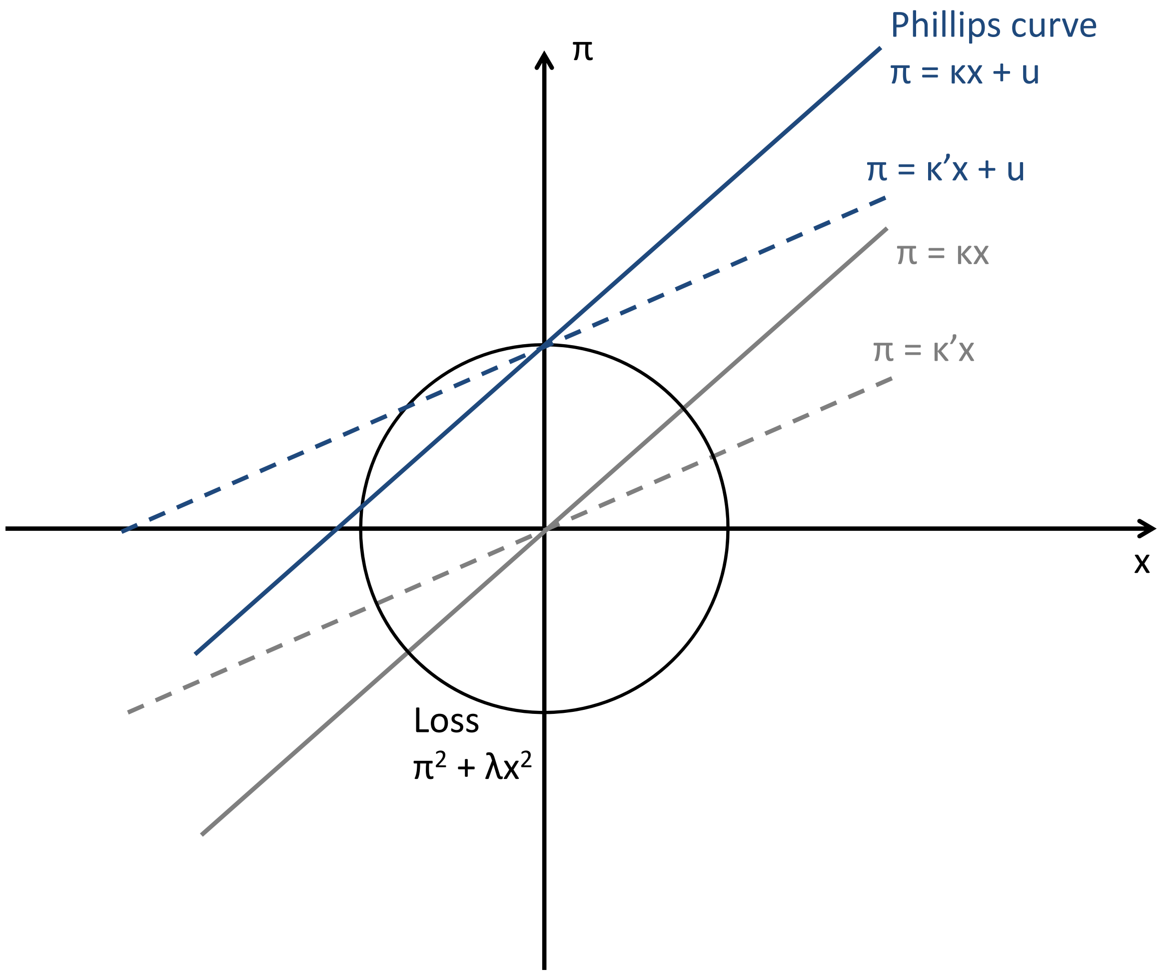

Figure 3: Graphical illustration of optimal monetary policy under discretion II

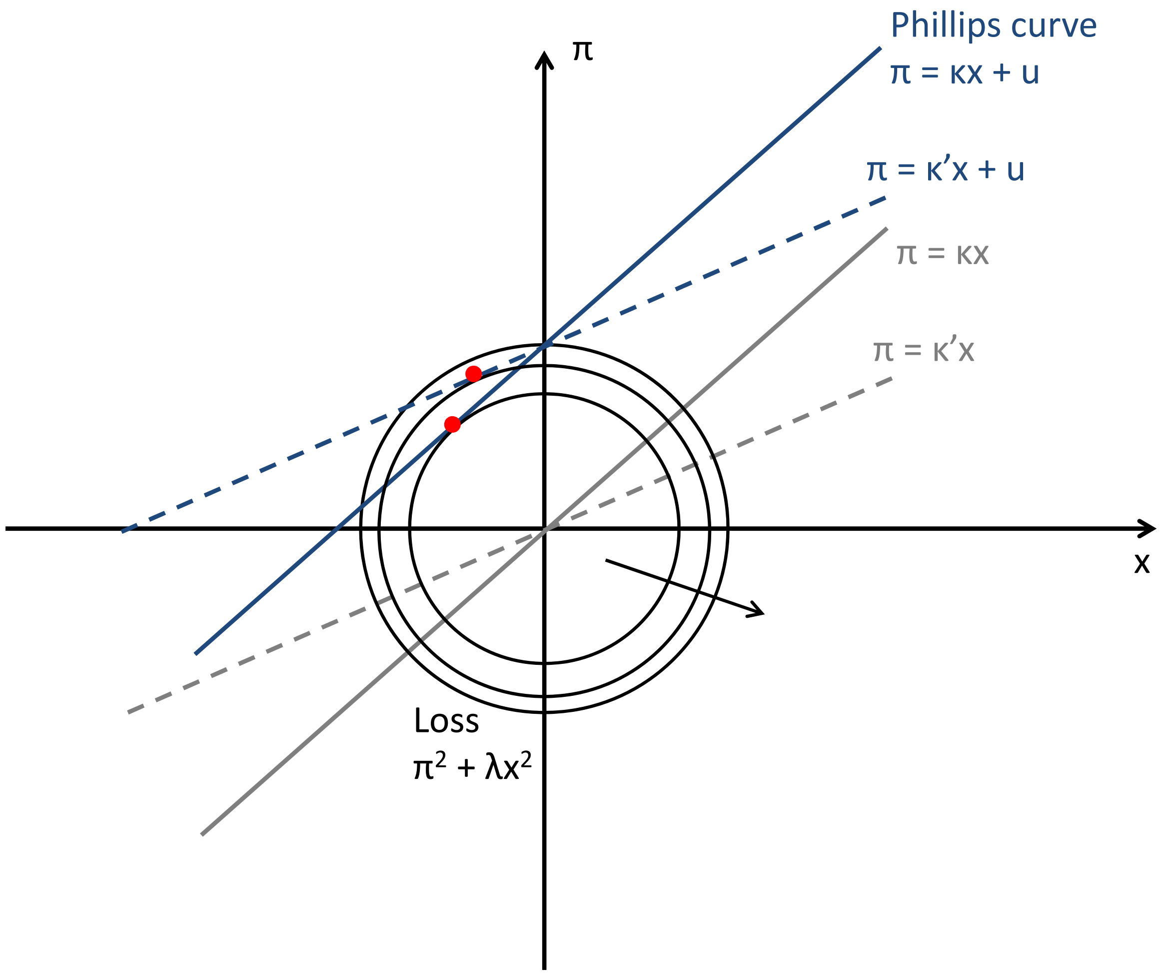

Figure 4: Graphical illustration of optimal monetary policy under discretion III

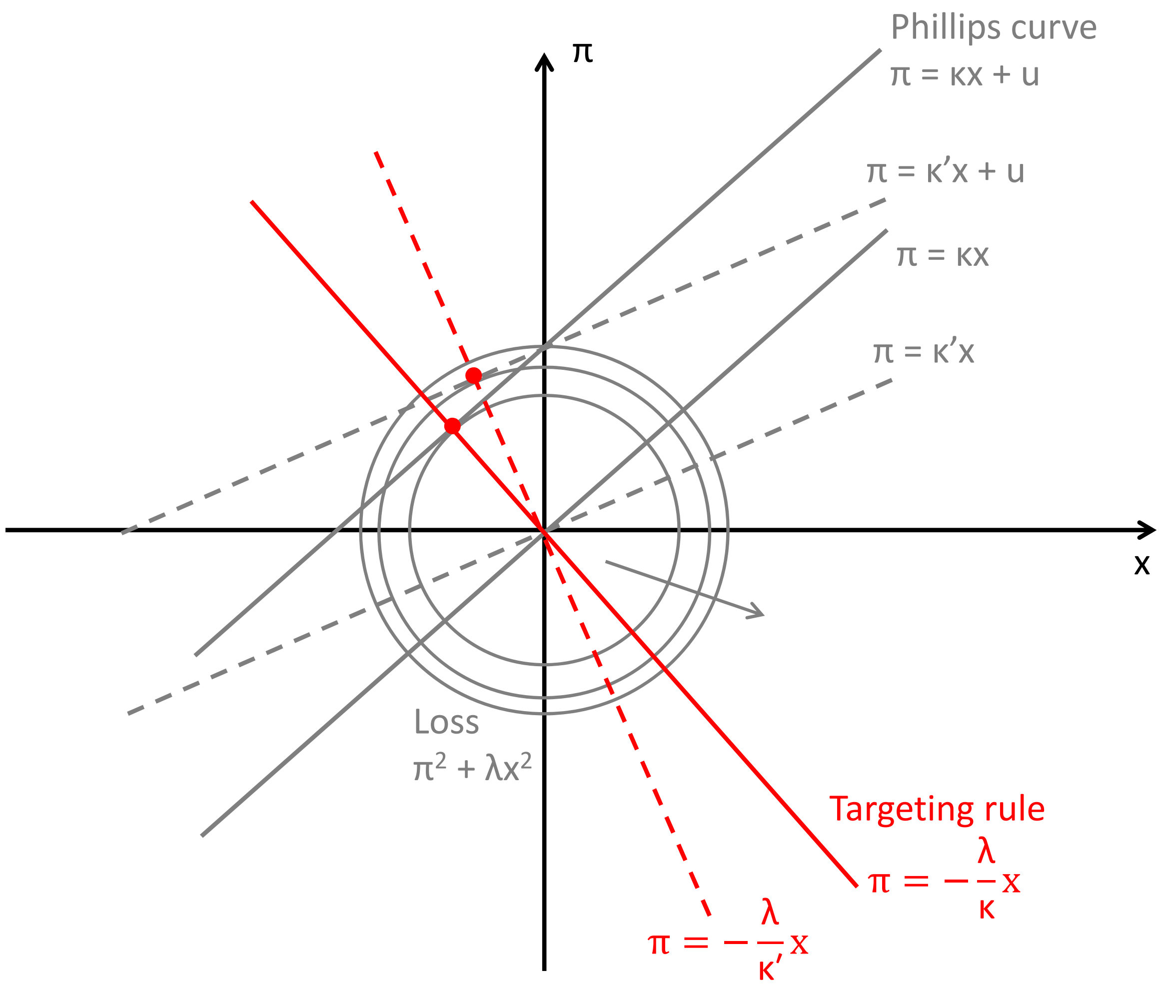

Figure 5: Graphical illustration of optimal monetary policy under discretion IV